

# Multiscale Technicolor and $b \rightarrow s\gamma$

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## Abstract

Correction to the  $b \rightarrow s\gamma$  branching ratio in the multiscale walking technicolor model (MWTCM) is examined. For the original MWTCM, the correction is too large to explain the recent CLEO data. We show that if topcolor is further introduced, the branching ratio in the topcolor assisted MWTCM can be in agreement with the CLEO data for a certain range of the parameters.

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# I. Introduction

Recently the CLEO collaboration has observed [1] the exclusive radiative decay  $B \rightarrow K^* \gamma$  with a branching fraction of  $BR(B \rightarrow K^* \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$ . The inclusive  $b \rightarrow s\gamma$  branching ratio measured by CLEO [2] is

$$BR(B \rightarrow X_s \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \quad (1)$$

The newest upper and lower limits of this decay branching ratio are

$$1.0 \times 10^{-4} < BR(B \rightarrow X_s \gamma) < 4.2 \times 10^{-4}, \text{ at } 95\% \text{C.L.} \quad (2)$$

As a loop-induced flavor changing neutral current ( FCNC ) process the inclusive decay ( at quark level )  $b \rightarrow s\gamma$  is in particular sensitive to contributions from those new physics beyond the standard model ( SM ) [3]. There is a vast interest in this decay.

The decay  $b \rightarrow s\gamma$  and its large leading log QCD corrections have been evaluated in the SM by several groups [4]. The reliability of the calculations of this decay is improving as partial calculations of the next- to-leading logarithmic QCD corrections to the effective Hamiltonian [5,6]

The great progress in theoretical studies and in experiments achieved recently encourage us to do more investigations about this decay in technicolor theories.

Technicolor ( TC ) [7] is one of the important candidates for the mechanism of naturally breaking the electroweak symmetry. To generate ordinary fermion mass, extended technicolor ( ETC ) [8] models have been proposed. The original ETC models suffer from the problem of predicting too large flavor changing neutral currents ( FCNC ). It has been shown, however, that this problem can be solved in walking technicolor ( WTC ) theories [9]. Furthermore, the electroweak parameter  $S$  in WTC models is smaller than that in the simple QCD-like ETC models and its deviation from the standard model ( SM ) value may fall within current experimental bounds [10]. To explain the large hierarchy of the quark masses, multiscale WTC models ( MWTCM ) are further proposed [11]. These models also predict a large number of interesting Pseudo-Goldstone bosons ( PGBs ) which are shown to be testable in future experiments [12]. So it is interesting to study physical consequences of these models.

In this paper, we examine the correction to the  $b \rightarrow s\gamma$  decay from charged PGBs in the MWTCM. We shall see that the original MWTCM gives too large correction to the

branching ratio of  $b \rightarrow s\gamma$  due to the smallness of the decay constant  $F_Q$  in this model. We shall show that if topcolor is further introduced, the branching ratio of  $b \rightarrow s\gamma$  in the topcolor assisted MWTCM can be in agreement with the CLEO data for a certain range of the parameters.

This paper is organized as the following: In Section II, we give a brief review of the MWTCM and then calculate the PGBs corrections to  $b \rightarrow s\gamma$  decay, together with the full leading log QCD corrections. In Section III, we obtain the branching ratio of this decay. The conclusions and discussions are also included in this Section.

## II. Charged PGBs of MWTCM and QCD

### corrections to $b \rightarrow s\gamma$

Let us start from the consideration of the MWTCM proposed by Lane and Ramana [11]. The ETC gauge group in this model is

$$G_{ETC} = SU(N_{ETC})_1 \times SU(N_{ETC})_2, \quad (3)$$

where  $N_{ETC} = N_{TC} + N_C + N_L$  in which  $N_{TC}$ ,  $N_C$  and  $N_L$  stand for the number of technicolors, the number of ordinary colors and the doublets of color-singlet technileptons, respectively. In Ref.[11],  $N_{TC}$  and  $N_L$  are chosen to be the minimal ones guaranteeing the walking of the TC coupling constant which are:  $N_{TC} = N_L = 6$ . The group  $G_{ETC}$  is supposed to break down to a diagonal ETC gauge group  $SU(N_{ETC})_{1+2}$  at a certain energy scale. The decay constant  $F_Q$  satisfies the following constraint [11]:

$$F = \sqrt{F_\psi^2 + 3F_Q^2 + N_L F_L^2} = 246 \text{GeV}. \quad (4)$$

It is found in Ref.[11] that  $F_Q = F_L = 20 - 40 \text{GeV}$ . We shall take  $F_Q = 40 \text{GeV}$  in our calculation. This present model predict the existence of a large number of PGBs, whose masses are typically expected to be larger than 100 GeV. In this paper, we shall take the mass of color-singlet PGBs  $m_{p^\pm} = 100 \text{ GeV}$  and the mass of color-octet PGBs  $m_{p_8^\pm} = (300 \sim 600) \text{ GeV}$  as the input parameters for our calculation.

The phenomenology of those color-singlet charged PGBs in the MWTCM is similar with that of the elementary charged Higgs bosons  $H^\pm$  of Type-I Two- Higgs-Doublet

model ( 2HDM ) [13]. And consequently, the contributions to the decay  $b \rightarrow s\gamma$  from the color-singlet charged PGBs in the MWTCM will be similar with that from charged Higgs bosons in the 2HDM. As for the color-octet charged PGBs, the situation is more complicated because of the involvement of the color interactions. Other neutral PGBs do not contribute to the rare decay  $b \rightarrow s\gamma$ .

The gauge coupling of the PGBs are determined by their quantum numbers. The Yukawa couplings of PGBs to ordinary fermions are induced by ETC interactions. The relevant couplings needed in our calculation are

$$[p^+ - -u_i - -d_j] = i \frac{1}{\sqrt{6} F_Q} V_{u_i d_j} [m_{u_i} (1 - \gamma_5) - m_{d_j} (1 + \gamma_5)], \quad (5)$$

$$[p_8^+ - -u_i - -d_j] = i \frac{1}{F_Q} V_{u_i d_j} \lambda^a [m_{u_i} (1 - \gamma_5) - m_{d_j} (1 + \gamma_5)], \quad (6)$$

where  $u = (u, c, t)$ ,  $d = (d, s, b)$ ;  $V_{u_i d_j}$  is the corresponding element of Kobayashi-Maskawa matrix;  $\lambda^a$  are the Gell-Mann  $SU(3)_c$  matrices.

In Fig.1, we draw the relevant Feynman diagram which contributes to the decay  $b \rightarrow s\gamma$ , where the blob represents the photonic penguin operators including the  $W$  gauge boson of the SM as well as the charged PGBs in the MWTCM. In the evaluation we at first integrate out the top quark and the weak  $W$  bosons at  $\mu = m_W$  scale, generating an effective five-quark theory. By using the renormalization group equation, we run the effective field theory down to b-quark scale to give the leading log QCD corrections, then at this scale, we calculate the rate of radiative b decay.

After applying the full QCD equations of motion [14], a complete set of dimension-6 operators relevant for  $b \rightarrow s\gamma$  decay can be chosen to be

$$\begin{aligned} O_1 &= (\bar{c}_{L\beta} \gamma^\mu b_{L\alpha}) (\bar{s}_{L\alpha} \gamma_\mu c_{L\beta}), \\ O_2 &= (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha}) (\bar{s}_{L\beta} \gamma_\mu c_{L\beta}), \\ O_3 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta} \gamma_\mu q_{L\beta}), \\ O_4 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta} \gamma_\mu q_{L\alpha}), \\ O_5 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta} \gamma_\mu q_{R\beta}), \\ O_6 &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta} \gamma_\mu q_{R\alpha}), \\ O_7 &= (e/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \\ O_8 &= (g/16\pi^2) m_b \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a. \end{aligned} \quad (7)$$

The effective Hamiltonian at the  $W$  scale is given as

$$H_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^8 C_i(m_W) O_i(m_W). \quad (8)$$

The coefficient of 8 operators are calculated and given as

$$\begin{aligned} C_i(m_W) &= 0, i = 1, 3, 4, 5, 6, C_2(m_W) = -1, \\ C_7(m_W) &= \frac{1}{2} A(x) + \frac{1}{3\sqrt{2}G_F F_Q^2} [B(y) + 8B(z)], \\ C_8(m_W) &= \frac{1}{2} C(x) + \frac{1}{3\sqrt{2}G_F F_Q^2} [D(y) + (8D(z) + E(z))], \end{aligned} \quad (9)$$

with  $x = (\frac{m_t}{m_W})^2$ ,  $y = (\frac{m_t}{m_{p\pm}})^2$ ,  $z = (\frac{m_t}{m_{p_8}})^2$ . The functions  $A$  and  $C$  arise from graphs with  $W$  boson exchange are already known contributions from the SM; while the functions  $B$ ,  $D$  and  $E$  arise from diagrams with color-singlet and color-octet charged PGBs of MWTCM. They are given as

$$\begin{aligned} A(x) &= -\frac{x}{12(1-x)^4} [(1-x)(8x^2 + 5x - 7) + 6x(3x - 2) \ln x], \\ B(x) &= \frac{x}{72(1-x)^4} [(1-x)(22x^2 - 53x + 25) + 6(3x^2 - 8x + 4) \ln x], \\ C(x) &= -\frac{x}{4(1-x)^4} [(1-x)(x^2 - 5x - 2) - 6x \ln x], \\ D(x) &= \frac{x}{24(1-x)^4} [(1-x)(5x^2 - 19x + 20) - 6(x - 2) \ln x], \\ E(x) &= -\frac{x}{8(1-x)^4} [(1-x)(12x^2 - 15x - 5) + 18x(x - 2) \ln x]. \end{aligned} \quad (10)$$

The running of the coefficients of operators from  $\mu = m_W$  to  $\mu = m_b$  was well described in Refs.[4]. After renormalization group running we have the QCD corrected coefficients of operators at  $\mu = m_b$  scale:

$$C_7^{eff}(m_b) = \varrho^{-\frac{16}{23}} [C_7(m_W) + \frac{8}{3} (\varrho^{\frac{2}{23}} - 1) C_8(m_W)] + C_2(m_W) \sum_{i=1}^8 h_i \varrho^{-a_i}, \quad (11)$$

with

$$\begin{aligned} \varrho &= \frac{\alpha_s(m_b)}{\alpha_s(m_W)}, \\ h_i &= (\frac{626126}{272277}, -\frac{56281}{51730}, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057), \\ a_i &= (\frac{14}{23}, \frac{16}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456). \end{aligned}$$

### III. The branching ratio of $B \rightarrow X_s \gamma$ and phenomenology

Following Refs.[4], applying the spectator model,

$$BR(B \rightarrow X_s \gamma) / BR(B \rightarrow X_c e \bar{\nu}) \approx \Gamma(b \rightarrow s \gamma) / \Gamma(b \rightarrow c e \bar{\nu}). \quad (12)$$

If we take experimental result  $BR(B \rightarrow X_c e \bar{\nu}) = 10.8\%$  [15], the branching ratio of  $B \rightarrow X_s \gamma$  is found to be

$$BR(B \rightarrow X_s \gamma) \approx 10.8\% \times \frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2} \frac{6\alpha_{QED}|C_7^{eff}(m_b)|^2}{\pi g(m_c/m_b)} (1 - \frac{2\alpha_s(m_b)}{3\pi} f(m_c/m_b))^{-1}. \quad (13)$$

Where the phase factor  $g(x)$  is given by

$$g(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x, \quad (14)$$

and the factor  $f(m_c/m_b)$  of one-loop QCD correction to the semileptonic decay is

$$f(m_c/m_b) = (\pi^2 - 31/4)(1 - m_c^2/m_b^2) + 3/2. \quad (15)$$

In numerical calculations we always use  $m_W = 80.22$  GeV,  $\alpha_s(m_Z) = 0.117$ ,  $m_c = 1.5$  GeV,  $m_b = 4.8$  GeV and  $|V_{tb}V_{ts}^*|^2/|V_{cb}|^2 = 0.95$  [15] as input parameters.

Fig.2 is a plot of the branching ratio  $BR(B \rightarrow X_s \gamma)$  as a function of  $m_{p_8^\pm}$  assuming  $m_t = 174$  GeV,  $m_{p^\pm} = 100$  GeV. The Long Dash line corresponds to the newest CLEO upper limit. From Fig.2 we can see that the contribution of the charged PGBs in the MWTCM is too large. In view of the above situation, we consider the topcolor assisted MWTCM. The motivation of introducing topcolor to MWTCM is the following: In the MWTCM, it is very difficult to generate the top quark mass as large as that measured in the Fermilab CDF and D0 experiments [16], even with strong ETC [17]. Thus, topcolor interactions for the third generation quarks seem to be required at an energy scale of about 1 TeV [18]. In the present model, topcolor is taken to be an ordinary asymptotically free gauge theory, while technicolor is still a walking theory for avoiding large FCNC [19]. As in other topcolor assisted technicolor theories [19], the electroweak symmetry breaking is driven mainly by technicolor interactions which are strong near 1 TeV. The ETC interactions give contributions to all quark and lepton masses, while the large mass of the top quark is mainly generated by the topcolor interactions introduced to the third generation quarks. The ETC-generated part of the top quark mass is  $m'_t = 66k$ , where  $k \sim 1$  to  $10^{-1}$  [19]. In this paper, we take  $m'_t = 15$  GeV and 20 GeV as input parameters in our calculation (i.e., in the above calculations, the  $m_t = 174$  GeV is substituted for  $m'_t = 15$  GeV and 20 GeV, the other input parameters and calculations are the same as the original MWTCM). The  $BR(B \rightarrow s\gamma)$  in topcolor assisted MWTCM is illustrated in Fig.3. From Fig.3 we

can see that the obtained  $BR(b \rightarrow s\gamma)$  is in agreement with the CLEO data for a certain range of the parameters.

In this paper, we have not considered the effects of other possible uncertainties, such as that of  $\alpha(m_Z)$ , next-to-leading-log QCD contribution [5], QCD correction from  $m_t$  to  $m_W$  [6] etc. The inclusion of those additional uncertainties will make the limitations weaken slightly.

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## Figure captions

Fig.1: The Feynman diagram which contributes to the rare radiative decay  $b \rightarrow s\gamma$ . The blob represents the photonic penguin operators including the  $W$  gauge boson of the SM as well as the charged PGBs in the MWTCM.

Fig.2: The plot of the branching ratio of  $b \rightarrow s\gamma$  versus the mass of charged color-octet PGBs  $m_{p_8^\pm}$  assuming  $m_t = 174$  GeV and the mass of color-singlet PGBs  $m_{p^\pm} = 100$  GeV in the MWTCM ( Solid line ). The Long Dash line corresponds to the newest CLEO upper limit.

Fig.3: The plot of the branching ratio of  $b \rightarrow s\gamma$  versus the mass of charged color-octet PGBs  $m_{p_8^\pm}$  assuming the color-singlet PGBs  $m_{p^\pm} = 100$  GeV in the topcolor assisted MWTCM. The Solid line represents the plot assuming  $m'_t = 15$  GeV, and the Dot Dash line represents the plot assuming  $m'_t = 20$  GeV. The Long Dash line and Short Dash line correspond the newest CLEO upper and lower limits, respectively.

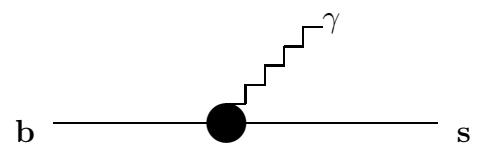


Fig.1